the identification of the TPC and the study of the quality of the estimates obtained enable obtaining more accurate and when necessary reliable information on the thermophysical properties of new and little-studied materials, which is especially important for the optimization and intensification of technological heat- and mass-transfer processes in metallurgy and machine building. The studies carried out enable posing the question of the simultaneous identification of several TPC and proceeding to combined IPH, in the course of whose solution the TPC and other conditions of uniqueness, for example, the boundary conditions, geometrical parameters, initial temperature distribution, and so on, are determined in parallel.

NOTATION

λ, coefficient of thermal conductivity; C_V, specific volume heat capacity; T_m, temperature of the medium; T_b and T_{in}, boundary and internal temperatues; α, heat-transfer coefficient) τ, time; $\Delta\tau$, time step; h, spatial step; q, heat flux, σ, rms deviation; and $\|\Delta Y\|$, total difference between the measured and predicted temperatures.

LITERATURE CITED

1. Yu. M. Matsevityi, A. V. Multanovskii, and M. Rekada, "Application of the method of optimal filtering to the identification of the coefficient of thermal conductivity," Énerget. Mashinostr., No. 38, 56-61 (1984).
2. Yu. M. Matsevityi and A. V. Multanovskii, Identification in Heat-Conduction Problems [in Russian], Naukova Dumka, Kiev (1982).
3. O. M. Alifanov, Identification of Heat Transfer Processes in Aircraft [in Russian], Mashinostroenie, Moscow (1979).
4. B. E. Neimark (ed.), Physical Properties of Steels and Alloys Used in Power Generation [in Russian], Énergiya, Moscow (1967).

NONLINEAR INVERSE PROBLEM OF RECONSTRUCTING TRANSPORT COEFFICIENTS

P. M. Kolesnikov and T. G. Protod'yakonova

UDC 536.24.01

The inverse coefficient problem for the quasilinear heat-conduction equation is solved numerically.

The experimental determination of thermophysical parameters is usually based on the solution of direct problems in the theory of heat conduction, when for fixed properties of the medium the temperature field is found with the help of the theory, and methods for confirming experimentally the theoretical results are created based on the theoretical representations. At high temperatures, however, experimental measurements are difficult to perform, so that the thermophysical properties of the materials are determined using the values of the temperature distribution measured far from the contact surface with the high-temperature flow. These problems, called inverse problems of transfer theory, have become very important in recent years in connection with the extensive possibilities presented by modern computational methods together with the extensive use of computers for rapid determination of the thermophysical parameters.

In most cases, linear mathematical models are used to solve inverse problems of determining the thermophysical parameters. In a wide range of temperature variation, however, the temperature dependence of the thermophysical parameters cannot be ignored, so that these methods obviously suffer from substantial errors. It is natural to base the experimental methods of determining thermophysical parameters on nonlinear mathematical models [1, 2].

In this work we study the problem of determining the nonlinear thermophysical characteristics, the heat capacity, and the coefficient of thermal conductivity for metal cylindrical samples interacting with a high temperature flow by the method of conjugate gradients.

A. V. Lykov Institute of Heat and Mass Transfer, Academy of Sciences of the Belorussian SSR, Minsk.

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 49, No. 6, pp. 909-915, December, 1985. Original article submitted May 17, 1985.

0022-0841/85/4906-1397$09.50

We shall write down the mathematical problem in the form of the following system of equations [1, 2, 3]:

$$c(T)\frac{\partial T}{\partial t}=\frac{1}{x^n}\frac{\partial}{\partial x}\left(x^n\lambda(T)\frac{\partial T}{\partial x}\right)+\mu(T)\frac{\partial T}{\partial x}+f(T),\ l_0<x<l_1,\ t>0, \tag{1}$$

$$T(x,\ 0)=T_0,\ l_0\leqslant x\leqslant l_1, \tag{2}$$

$$A_1T+A_2\lambda(T)\frac{\partial T}{\partial x}=A_3(T),\ x=l_0,\ t>0, \tag{3}$$

$$B_1T-B_2\lambda(T)\frac{\partial T}{\partial x}=B_3(T),\ x=l_1,\ t>0 \tag{4}$$

and additional conditions

$$T(x_i,\ t)=\varphi_i(t),\ i=1,\ \ldots,\ m,$$

where $\varphi_i(t)$, $\mu(T)$, $f(T)$, $A_3(T)$, $B_3(T)$ are known functions. The functions $c(T)$ and $\lambda(T)$ were approximated by expressions of the form

$$c(T)=\sum_{i=0}^{I}c_iT^{k_i},\ \lambda(T)=\sum_{l=0}^{L}\lambda_lT^{k_l'}.$$

We define the parameters c_i, λ_i, written for convenience in the form of the components of a vector $\bar{a}=(c_0,\ \ldots,\ c_I,\ \lambda_0,\ \ldots,\ \lambda_L)$, with the help of the minimization of the discrepancy functional [4-6]:

$$J(\bar{a})=\sum_{i=1}^{m}\int_0^{t_1}\beta_i(t)(T(x_i,\ t)-\varphi_i(t))^2\,dt,$$

characterizing the matching of the measured and predicted values of the temperatures, where $\beta_i(t)$ are given positive functions, determining the degree of reliability of the additional information. These functions are chosen to be different depending on the error in the experiment.

We minimized the functional $J(\bar{a})$ by the method of conjugate gradients. We write down the components of the gradient of the functional $J(\bar{a})$ in terms of the values of the sensitivity function $\psi_j=\partial T/\partial a_j$:

$$\frac{\partial J}{\partial a_j}=2\sum_{i=1}^{m}\int_0^{t_1}\beta_i(t)(T(x_i,\ t)-\varphi_i(t))\,\psi_j(x_i,\ t)\,dt. \tag{5}$$

The functions $\psi_j(x,\ t)$, $j=1,\ \ldots,\ I+L+2$ are the solutions of the systems of equations

$$\begin{aligned}c(T)\frac{\partial\psi_j}{\partial t}&=\frac{1}{x^n}\frac{\partial}{\partial x}\left(x^n\lambda(T)\frac{\partial\psi_j}{\partial x}\right)+\mu(T)\frac{\partial\psi_j}{\partial x}+\\&+\frac{1}{x^n}\frac{\partial}{\partial x}\left(x^n\frac{\partial\lambda}{\partial T}\frac{\partial T}{\partial x}\psi_j\right)+\left(\frac{\partial\mu}{\partial T}\frac{\partial T}{\partial x}+\frac{\partial f}{\partial x}-\frac{\partial c}{\partial T}\frac{\partial T}{\partial t}\right)\psi_j+\\&+\frac{1}{x^n}\frac{\partial}{\partial x}\left(x^n\frac{\partial\lambda}{\partial a_j}\frac{\partial T}{\partial x}\right)-\frac{\partial c}{\partial a_j}\frac{\partial T}{\partial t},\ l_0<x<l_1,\ t>0\end{aligned} \tag{6}$$

with the following conditions:

initial

$$\psi_j(x,\ 0)=0,\ l_0\leqslant x\leqslant l_1 \tag{7}$$

and boundary

$$\left(A_1 + A_2 \frac{\partial\lambda}{\partial T}\frac{\partial T}{\partial x}\right)\psi_j + A_2\lambda(T)\frac{\partial\psi_j}{\partial x} + A_2\frac{\partial\lambda}{\partial a_j}\frac{\partial T}{\partial x} = \frac{\partial A_3}{\partial T}\psi_j,\ x = l_0,\ t > 0, \tag{8}$$

$$\left(B_1 - B_2 \frac{\partial\lambda}{\partial T}\frac{\partial T}{\partial x}\right)\psi_j - B_2\lambda(T)\frac{\partial\psi_j}{\partial x} - B_2\frac{\partial\lambda}{\partial a_j}\frac{\partial T}{\partial x} = \frac{\partial B_3}{\partial T}\psi_j,\ x = l_1,\ t > 0. \tag{9}$$

The problem is solved numerically. The difference analogs of the equations (1)-(9), approximating them with an error of $o(\tau+h^2)$, are

$$c_i T_{\bar{t},i} = \frac{1}{x^n}(x^n\lambda T_{\bar{x}})_{\hat{x}} + \mu_j T_{\overset{\circ}{x}} + f_i, \tag{10}$$

$$T_i^0 = T_0, \tag{11}$$

$$A_1 T_0 + A_2(\lambda_{1/2}T_{x,0} - 0{,}5h(c_0 T_{\bar{t},0} - f_0)/n')/M_0 = A_{3,0}, \tag{12}$$

$$B_1 T_N - B_2(\lambda_{N-1/2}T_{\bar{x},N} + 0{,}5h(c_N T_{\bar{t},N} - f_N))/M_N = B_{3,N}, \tag{13}$$

$$c\psi_{j\,\bar{t}} = \frac{1}{x^n}(x^n\lambda\psi_{j\,\bar{x}})_{\hat{x}} + \mu\psi_{j\overset{\circ}{x}} + \frac{1}{x^n}\left(x^n\frac{\partial\lambda}{\partial T}T_{\bar{x}}\psi_j\right)_{\hat{x}} +$$

$$+\left(\frac{\partial\mu}{\partial T}T_{\overset{\circ}{x}} + \frac{\partial f}{\partial T} - \frac{\partial c}{\partial T}\frac{T_{\bar{t}}}{\partial t}\right)\psi_j + \frac{1}{x^n}\left(x^n\frac{\partial\lambda}{\partial a_j}T_{\bar{x}}\right)_{\hat{x}} - \frac{\partial c}{\partial a_j}T_{\bar{t}}, \tag{14}$$

$$\psi_j^0 = 0, \tag{15}$$

$$\left(A_1 + A_2\frac{\partial\lambda}{\partial T}T_{x,0} - \frac{\partial A_3}{\partial T}\right)\psi_{j,0} + A_2\frac{\partial\lambda}{\partial a_j}T_{x,0} + A_2\lambda_{1/2}\psi_{jx,0} = 0, \tag{16}$$

$$\left(B_1 - B_2\frac{\partial\lambda}{\partial T}T_{\bar{x},N} - \frac{\partial B_3}{\partial T}\right)\psi_{j,N} - B_2\frac{\partial\lambda}{\partial a_j}T_{\bar{x},N} - B_2\lambda_{N-1/2}\psi_{j\,\bar{x},N} = 0, \tag{17}$$

$$J = \sum_{i=1}^{m}\sum_{k=1}^{K}\beta_i^k(T_{n_i}^k - \varphi_i^k)^2\tau, \tag{18}$$

$$\frac{\partial J}{\partial a_j} = 2\sum_{i=1}^{m}\sum_{k=1}^{K}\beta_i^k(T_{n_i}^k - \varphi_i^k)\psi_{jn_i}^k\,\tau, \tag{19}$$

where

$$n' = \begin{cases} n+1, & l_0 = 0, \\ 1, & l_0 \neq 0, \end{cases} \quad ,\ n_i:\ n_i h = x_i,\ \tau K = t_i;$$

$$M_0 = \begin{cases} 1 - 0.5\,h\,\mu_0/\lambda_0\,(1+n), & l_0 = 0; \\ 1 - 0.5\,h\,\mu_0/\lambda_0 - 0.5\,hn/l_0, & l_0 \neq 0; \end{cases}$$

$$M_N = 1 + 0.5\,h\,\mu_N/\lambda_N + 0.5\,hn/l_1.$$

To determine the parameters $c_0, \ldots, c_I, \lambda_0, \ldots, \lambda_L$, the components of the vector $\bar{a}$, using the method of conjugate gradients we shall construct the following iteration process: if the value of $\overset{s}{\bar{a}}$, is known, then, by solving the difference equations (10)-(17), we determine $\overset{(s)}{T_i^k}$, $\overset{(s)}{\psi_{ji}^k}$, and in terms of them we calculate $\overset{(s)}{J}$ and grad $\overset{(s)}{J}$ from the formulas (18) and (19). We shall seek the next (s + 1)-st approximation in the form

TABLE 1. Predicted $T(x_i, t)$ and Experimental $\varphi(x_i, t)$ Temperatures at Different Times t and at Different Coordinates x_i

t, c	$\varphi_1(t)$	$T(x_1, t)$	$\varphi_2(t)$	$T(x_2, t)$	$\varphi_3(t)$	$T(x_3, t)$
0,05	20,32	20,32	20,01	20,006	20,00	20,00
0,10	21,19	21,19	20,03	20,0297	20,00	20,00
0,15	22,75	22,74	20,09	20,084	20,00	20,002
0,20	25,03	25,03	20,22	20,183	20,01	20,005
0,25	28,05	28,05	20,43	20,346	20,02	20,012
0,3	31,78	31,78	20,74	20,59	20,04	20,023
0,35	36,17	36,17	21,19	20,934	20,07	20,04
0,4	41,19	41,19	21,79	21,397	20,12	20,067
0,45	46,77	46,77	22,54	22,006	20,19	20,10
0,5	52,87	52,87	23,47	22,779	20,28	20,156
0,55	59,44	59,44	24,59	23,741	20,41	20,226
0,6	66,43	66,43	25,90	24,916	20,57	20,318
0,65	73,81	73,81	27,40	26,329	20,78	20,437
0,7	81,54	81,54	29,09	29,003	21,04	20,59
0,75	89,58	89,58	30,98	29,964	21,34	20,781
0,8	97,91	97,91	33,06	32,234	21,71	21,019
0,85	106,50	106,50	35,34	34,835	22,14	21,313
0,9	115,33	115,33	37,80	37,798	22,63	21,672
0,95	124,38	124,38	40,44	41,11	23,18	22,106
1,0	133,62	133,62	43,26	44,82	23,81	22,63

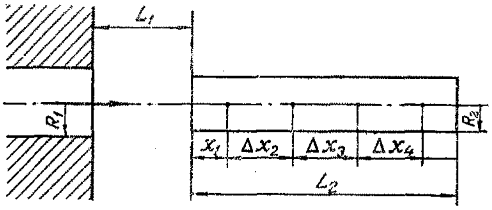

Fig. 1. Diagram of the arrangement of thermocouples along the probe: $R_1 = 0.004$; $R_2 = 0.003$; $L_1 = 0.015$; $L_2 = 0.04$; $x_1 = 0.005$; $\Delta x_2 = x_2 - x_1 = 0.01$; $\Delta x_3 = x_3 - x_2 = 0.01$; $\Delta x_4 = x_4 - x_3 = 0.01$.

$$\overset{(s+1)}{\underline{a}} = \overset{(s)}{\underline{a}} - \overset{(s)}{\alpha}\,\overset{(s)}{\underline{p}},$$

where

$$\overset{(s)}{\underline{p}} = \operatorname{grad} \overset{(s)}{J} - \overset{(s)}{\beta}\,\overset{(s-1)}{\underline{p}};\quad \overset{(s)}{\beta} = -\frac{|\operatorname{grad} \overset{(s)}{J}|^2}{|\operatorname{grad} \overset{(s-1)}{J}|^2};\quad \overset{(0)}{\underline{p}} = \operatorname{grad} \overset{(0)}{J};$$

the descent parameter $\overset{(s)}{\alpha}$ is chosen so that

$$J(\overset{(s+1)}{\underline{a}}) < J(\overset{(s)}{\underline{a}}).$$

The iteration process terminates when the condition $J < \delta$, where δ is a fixed error, is satisfied.

To check the efficiency of the proposed algorithm and the corresponding program, we carried out a control calculation. From the solution of the direct problem with given thermophysical coefficients

$$c(T) = c_0 + c_1 T = 3800 + 2T,\ \lambda(T) = \lambda_0 + \lambda_1 T = 0.35 - 0.00001\,T,$$
$$\mu(T) = 0,\ f(T) = 0,$$

with the boundary conditions

$$\lambda(T)\,\partial T/\partial x = -10\,000\sqrt{4t} \text{ at } x = 0,$$

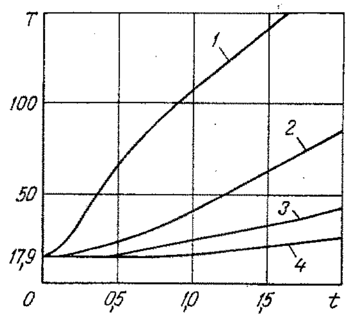

Fig. 2. Distribution of the temperature T as a function of time: 1) x = 0.005; 2) 0.015; 3) 0.025; 4) 0.035. T, °C; t, sec.

TABLE 2. Comparison of the Refined Predicted T and Experimental φ Temperatures

t, sec	$\varphi_1(t)$	$\varphi_2(t)$	$T(x_2,t)$	$T(x_2,t)$	$\varphi_3(t)$	$T(x_3,t)$	$T(x_3,t)$	$\varphi_4(t)$	$T(x_4,t)$	$T(x_4,t)$
0,1	21,45	17,93	18,09	18,09	17,93	17,94	17,93	17,93	17,93	17,93
0,2	36,41	18,29	19,05	19,05	17,93	17,99	17,99	17,93	17,93	17,93
0,3	50,75	19,81	21,05	21,04	17,93	18,17	18,17	17,93	17,95	17,94
0,4	63,86	23,20	23,99	23,98	18,52	18,53	18,53	17,93	17,98	17,98
0,5	75,63	26,67	27,67	27,65	19,11	19,13	19,12	17,93	18,04	18,04
0,6	85,80	31,28	31,82	31,8	20,28	19,94	19,98	17,93	18,15	18,15
0,7	95,10	36,18	36,26	36,24	20,86	21,10	21,09	18,17	18,32	18,31
0,8	104,54	40,48	40,90	40,88	22,15	22,46	22,44	18,64	18,55	18,54
0,9	112,75	45,42	45,65	45,63	23,78	24,04	24,02	18,87	18,85	18,84
1,0	120,47	50,42	50,43	50,42	25,52	25,81	25,79	19,22	19,22	19,21
1,1	127,50	55,37	55,20	55,18	27,48	27,74	27,72	20,04	19,66	19,65
1,2	134,75	59,96	59,93	59,92	29,55	29,81	29,79	20,63	20,16	20,15
1,3	141,44	64,62	64,63	64,62	31,85	31,99	31,97	21,45	20,72	20,71
1,4	147,97	69,09	69,27	69,27	34,13	34,27	34,25	22,61	21,34	21,32
1,5	154,15	72,87	73,85	73,86	36,96	36,62	36,59	23,78	21,99	21,88
1,6	160,28	77,95	78,37	78,39	39,35	39,02	38,99	24,94	22,69	22,68
1,7	165,80	82,68	82,82	82,83	40,93	41,45	41,44	26,56	23,41	23,40
1,8	171,82	86,84	87,20	87,22	43,63	43,92	43,91	27,83	24,18	24,16
1,9	177,34	90,47	91,52	91,55	47,09	46,41	46,40	29,56	24,94	24,93
2,0	182,45	94,58	95,76	95,80	49,86	48,91	48,90	30,93	25,73	25,72

$$\lambda(T)\,\partial T/\partial x = 0 \quad \text{at} \quad x = 1$$

and the initial temperature $T(x, 0) = 20°C$ we obtained for the case $n = 0$ the values of $\varphi_i(t)$ — the change in the temperature as a function of time at the points $x_1 = 0.005$, $x_2 = 0.01$, $x_3 = 0.015$. Then we solved the inverse problem, in which the functions $\varphi_i(t)$ with different initial approximations c_0 and λ_0 were used as the initial information. In solving it with $c_0 = 4000$ and $\lambda_0 = 0.4$ the following dependences were obtained:

$$c(T) = 4000 + 0{,}15 \cdot 10^{-6}\,T,\ \lambda(T) = 0.4 - 0.00188\,T.$$

In this case the value of the discrepancy function J is equal to 0.93, while its gradient

$$\left\{ \frac{\partial J}{\partial c_0};\ \frac{\partial J}{\partial c_1};\ \frac{\partial J}{\partial \lambda_0};\ \frac{\partial J}{\partial \lambda_1} \right\}$$

contains components equal to {−0.0073; 0.146; 75.98; 1200.44}. The quantity $\partial J/\partial \lambda_1$ has in this case the highest value and in calculating the approximation $\overset{(s+1)}{a} = (\overset{(s+1)}{c_0},\ \overset{(s+1)}{c_1},\ \overset{(s+1)}{\lambda_0},\ \overset{(s+1)}{\lambda_1})$ in order that the value of the discrepancy J decreases while the functions $c(T) = c_0 + c_1 T$ and $\lambda(T) = \lambda_0 + \lambda_1 T$ would remain positive, the descent parameter $\overset{(s)}{\alpha}$ is selected to be small enough that $\overset{(s+1)}{c_0}$ and $\overset{(s+1)}{\lambda_0}$ actually do not change, but they may not correspond to the true values of c_0 and λ_0. To eliminate this deficiency and taking into account the fact that $\partial J/\partial c_0$ assumes the smallest value, it is necessary to determine as accurately as possible the first approximation for c_0. A more accurate value of λ_0 is obtained by solving the inverse problem with constant coefficient $c(T) = c_0$ and $\lambda(T) = \underline{\lambda}_0$, after which the problem (10)-(19) is solved once again with the more accurate parameters $\bar{c}_0 = 3800$ and $\bar{\lambda}_0 = 0.3$ taking into account the temperature dependence of c and λ. Choosing, for example, $\bar{\lambda} = 0.365$, in solving the problem with constant thermophysical parameters, we obtain the value λ, while the discrepancy functional is equal to 36.76. We therefore continued the calculations taking into account the temperature dependence of c and λ, and the functions

$$c(T) = 3800 + 0.362 \cdot 10^{-6} T,$$
$$\lambda(T) = 0.3653 + 0.012\,T$$

were determined with 21 iterations, the discrepancy J decreased in this case to 0.746.

The predicted temperatures at the control points with the additional values $\varphi_i(t)$ are compared in Table 1.

With the help of this algorithm we reconstructed the heat capacity and coefficient of thermal conductivity of a copper cylindrical sample. For the auxiliary data we used the experimentally determined temperature at several points in the volume of a calorimetric probe [3, 7, 8]. The heating occurs on the end face of a probe (Fig. 1) inserted into the housing of the calorimeter. The heat flow in the probe is one-dimensional, and the cylindrical surface of the probe is insulated by an air layer. During the experiment the temperature (Fig. 2) was measured at four points along the cylinder (Fig. 1). Since the exact determination of the heat flux at the boundary is also a laborious and difficult problem, in solving the inverse problem of reconstructing the thermophysical parameters at the point x = 0.005 boundary conditions of the first kind were given: the temperature at this point was assumed to be equal to the experimentally measured temperature; a graph of its variation as a function of time is presented in Fig. 2. On the right boundary (x = 0.04) the condition of zero heat flux

$$\lambda \frac{\partial T}{\partial x} = 0$$

was given. Initially the temperature of the sample is equal to 17.93°C.

In fixing the initial approximations $c_0 = 3400$ and $\lambda_0 = 0.4$ for the thermophysical characteristics the following dependences were obtained [9]:

$$c(T) = 3400 + 0.79 \cdot 10^{-9}\,T,$$
$$\lambda(T) = 0.362 - 0.236 \cdot 10^{-4} T.$$

The value of the discrepancy functional J in this case is equal to 9.37.

In the second variant of the initial approximations $c_0 = 3800$, $\lambda_0 = 0.35$ the reconstructed functions c(T), λ(T) have the form

$$c(T) = 3800 - 0.1 \cdot 10^{-8} T,$$
$$\lambda(T) = 0.4033 - 0.1454 \cdot 10^{-4} T,\ J = 9.422.$$

The predicted and experimental values of the temperatures are compared in Table 2.

Columns 1, 2, 5, and 8 show the experimentally measured values of the temperatures at points with the coordinates $x_0 = 0.005$; $x_1 = 0.015$; $x_2 = 0.025$; $x_3 = 0.035$, respectively. Columns 3, 6, and 9 show the predicted temperature distribution with $c(T) = 3400 + 0.79 \cdot 10^{-9}$ T, $\lambda(T) = 0.362 - 0.236 \cdot 10^{-4}$ T at the points x_1, x_2, x_3, columns 4, 7, and 10 show the predicted temperature distribution with $c(T) = 3800 - 0.1 \cdot 10^{-8}$ T, $\lambda(T) = 0.4033 - 0.1454 \cdot 10^{-4}$ T for the same values of x, respectively.

If the data for copper are approximated by the expressions c(T) = 3382, $\lambda(T) = 0.3984 - 0.0000457$ T, then it is evident that the reconstructed characteristics from the solution of the inverse problem give a good approximation.

NOTATION

T, temperature; T_0, initial temperature; $x(l_0 \leq x \leq l_1)$, spatial coordinate; $t(0 \leq t \leq t_1)$, time; c(T), volumetric heat capacity; $\lambda(T)$, coefficient of thermal conductivity; A_1, A_2, B_1, B_2, coefficients equal to 0 or 1 depending on the form of the boundary condition; $\varphi_i(t)$, measured values of the temperature at the points x_i; J, discrepancy functional; $\beta_i(t)$, weight functions; and ψ_j, sensitivity function.

LITERATURE CITED

1. P. M. Kolesnikov, Methods of Transport Theory in Nonlinear Media [in Russian], Nauka i Tekhnika, Minsk (1981).
2. P. M. Kolesnikov and L. V. Grishanova, "Study of the propagation of thermal fields in nonlinear media," in: Analytical and Numerical Methods in Transport Theory [in Russian], Institute of Heat and Mass Transfer, Academy of Sciences of the Belorussian SSR, Minsk (1977).
3. P. M. Kolesnikov, R. T. Abramov, V. P. Veselov, et al., "Direct and inverse problems of nonlinear heat conduction accompanying the interaction of a high-temperature flow with copper stands," in: Heat- and Mass Transfer VII [in Russian], Vol. 7, Minsk (1984), pp. 93-97.
4. O. M. Alifanov, Identification of Heat Transfer Processes in Aircraft (Introduction to the Theory of Inverse Problems) [in Russian], Mashinostroenie, Moscow (1979).
5. E. A. Artyukhin, "Reconstruction of the coefficient of thermal conductivity from the solution of a nonlinear inverse problem," Inzh.-Fiz. Zh., 41, No. 4, 587-592 (1981).
6. O. M. Alifanov, E. A. Artyukhin, and B. M. Pankratov, "Solution of a nonlinear inverse problem of thermal conductivity," in: Heat and Mass Transfer V [in Russian], Vol. 9, Minsk (1976), pp. 94-103.
7. R. T. Abramov, P. M. Kolesnikov, L. Kreichi, and I. Fogel, "Inverse problem of the determination of the nonlinear dependences of thermophysical properties of materials and heat flows during flow of high-temperature gas over surfaces of bodies," in: Energy Transfer in Nonlinear, Inhomogeneous and Nonequilibrium Media [in Russian], Minsk (1984), pp. 59-67.
8. L. Kreichi and I. Fogel, "Numerical processing of nonstationary calorimetric measurements," in: Heat and Mass Transfer [in Russian], Institute of Heat and Mass Transfer, Academy of Sciences of the Belorussian SSR, Vol. 9, Pt. 2, Minsk (1972), pp. 498-504.
9. V. S. Chirkin, Thermophysical Properties of Nuclear Engineering Materials [in Russian], Atomizdat, Moscow (1968).